thermodynamics may be used in addition to perfect gas In this regard, the characteristics approach offers a distinct advantage over any approximate method which assumes a perfect gas

It is felt that both approaches have merit, and their use is dependent upon the accuracy of the results desired and the mathematical tools available

Comment on Calculation of Oblique Shock Waves

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Nomenclature

M = Mach number upstream from an oblique shock wave
θ = acute angle between an oblique shock wave and the free-stream flow direction
δ = acute angle between a deflecting surface and the free-stream flow direction
γ = ratio of specific heats

Many authors have pointed out the lack of simple relationships connecting the deflection angle δ and the flow across a resulting oblique shock wave [1–3]

Since most problems are specified by a deflection angle, it is necessary then to find the shock wave angle θ corresponding to the flow conditions and the specified deflection The calculation of the fluid-property changes across the shock wave, knowing the wave angle, is a simple and straightforward problem [2 4] Recognizing the need for a procedure to obtain the wave angle in terms of the parameters specifying the problem, Thompson[1] presented a cubic in $\sin^2\theta$, which reduces to the following:

$$\sin^6\theta + a\sin^2\theta + b = 0$$

where

$$a = \frac{2M^2+1}{M^4} + \left[\frac{(\gamma+1)^2}{4} + \frac{\gamma-1}{M^2}\right]\sin^2\delta - \frac{k^2}{3} \tag{1}$$

$$b = -\frac{2}{27}k^3 + \frac{k}{3}\left\{\frac{2M^2+1}{M^4} + \left[\frac{(\gamma+1)^2}{4} + \frac{\gamma-1}{M^2}\right]\sin^2\delta\right\} - \frac{\cos^2\delta}{M^4} \tag{2}$$

$$k = \frac{M^2+2}{M^2} + \gamma\sin^2\delta \tag{3}$$

Thompson further suggested that numerical procedures be used to derive the roots of this equation to any desired degree of accuracy Numerical procedures are not required, however, and the roots can be obtained in closed form The Cardan equations[4] give three roots for $\sin^2\theta$ from the foregoing equation The smallest of these roots corresponds to an entropy decrease and must be discarded immediately as being a violation of the basic laws of thermodynamics The largest root corresponds to the strong shock wave solution, and the remaining root, the case of primary interest, is the weak shock solution The weak shock root is given by

$$\sin^2\theta = -\frac{A+B}{2} - \frac{A-B}{2}(-3)^{1/2} + \frac{k}{3} \tag{4}$$

where

$$A = \left[-\frac{b}{2} + \left(\frac{b^2}{4} + \frac{a^3}{27}\right)^{1/2}\right]^{1/3} \tag{5}$$

$$B = \left[-\frac{b}{2} - \left(\frac{b^2}{4} + \frac{a^3}{27}\right)^{1/2}\right]^{1/3} \tag{6}$$

Employing the Argand representation of complex quantities and Euler's relationship, this root further reduces to

$$\sin^2\theta = -2\left(-\frac{a}{3}\right)^{1/2}\left\{\cos\left[\frac{\tan^{-1}(-2h/b)}{3}\right] + (3)^{1/2}\sin\left[\frac{\tan^{-1}(-2h/b)}{3}\right]\right\} + \frac{k}{3} \tag{7}$$

where

$$h = [(-b^2/4) - (a^3/27)]^{1/2} \tag{8}$$

Manipulation of the trigonometric functions gives as the final result

$$\sin^2\theta = \frac{k}{3} - 2\left(-\frac{a}{3}\right)^{1/2}\cos\left[60^\circ + \frac{\tan^{-1}(-2h/b)}{3}\right] \tag{9}$$

Similarly, the root corresponding to the strong wave reduces to

$$\sin^2\theta = \frac{k}{3} + 2\left(-\frac{a}{3}\right)^{1/2}\cos\left[\frac{\tan^{-1}(-2h/b)}{3}\right] \tag{10}$$

Thus, the properties across an oblique shock wave arising from a given deflection and flow condition can be determined in closed form It requires merely the substitution of the given conditions into Eqs (9) and (10) to determine the shock wave angle This angle is then substituted into the easily derivable oblique shock relationships, which are widely available in the literature [2 3]

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Received January 27, 1963
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Comment on "Application of Dynamic Programming to Optimizing the Orbital Control Process of a 24-Hour Communications Satellite"

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Reference 1 treats the minimization of the sum of the squares of the terminal errors in three orbital parameters subject to a constraint on the amount of fuel consumed

Received December 20, 1963; revision received February 14, 1964
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Unfortunately, this constraint is not a direct measure of the fuel consumed and leads to a thrust program quite different from the optimum program with a fixed amount of fuel

The constraint used is the sum of the squares of the velocity impulses, whereas the fuel consumption depends upon the sum of the absolute magnitudes of the velocity impulses For a fixed value of the constraint, the sum of the absolute magnitudes must lie somewhere between the square root of the sum of the squares of the impulses and the square root of the number of impulses multiplied by the square root of the sum of the squares of the impulses The particular value of the fuel consumption for a given value of the constraint depends upon the distribution of the impulses

An analytic solution has been presented for maximizing the changes in the three orbital parameters considered in Ref 1 with a fixed amount of fuel [2-4] The solution of this more practical problem leads to impulses applied only at the line of nodes The solution obtained numerically with this particular constraint on fuel consumption leads to impulses distributed all around the orbit and is quite different from the analytic solution with a given amount of fuel

The value of this particular constraint on fuel consumption appears to be rather limited

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Comments on "Radiation Slip"

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THE purpose of this note is to suggest an improvement of the method that appears in Ref 1 and to point out some of the limitations of this approach

In the radiation problem, the fourth power of the temperature plays the analogous role to the first power of the velocity in the normal viscous diffusion of momentum or the first power of the temperature in the thermal conduction of heat This analogy is obvious if Eqs (4) and (5) of Ref 1 are combined to give

$$-q = \tfrac{4}{3}\sigma l(dT^4/dy) = \tfrac{4}{3}\sigma(dT^4/d\tau)$$

using the notation of Ref 1 The analogous slip condition becomes

$$T^4 - T_w{}^4 = Kl(dT^4/dy) = K(dT^4/d\tau)$$

This condition gives Eq (9) of Ref 1 for all values of slip, whereas Eq (8) of Ref 1 does not, except in the limit of small slip Since Eq (8) is not used again, but all of the results are based on Eq (9), this change does not affect the results presented except that it may make the conclusion that Eq (11) gives good results even for large slip a little more reasonable

Received January 8, 1964

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However, difficulty is encountered if Eq (8) is used to calculate the temperature jump at the wall, but this difficulty is eliminated by the suggested improvement A comparison of the temperature profiles given by the slip method using the improved boundary conditions and the numerical calculations of Ref 2 is shown in Fig 1 The agreement of the suggested method with the numerical calculations is quite good, whereas using Eq (8) to calculate the temperature jump at the wall will cause large errors except for very small jumps

It also is useful to point out some of the limitations of this method In the example chosen, the method was applied to a Couette problem, where good agreement with exact results was obtained similar to the situation for the conduction of heat through a low-density gas Difficulty has been encountered in applying these methods directly to more complicated problems, such as the Rayleigh problem, for low-density gas flow, and somewhat similar difficulties occur for radiation The difficulty in the case of radiation is connected with the equating of the photon and gas temperature In the absence of conduction, the gas temperature is unimportant in the Couette problem, since the gas is not absorbing or giving off heat This condition does not

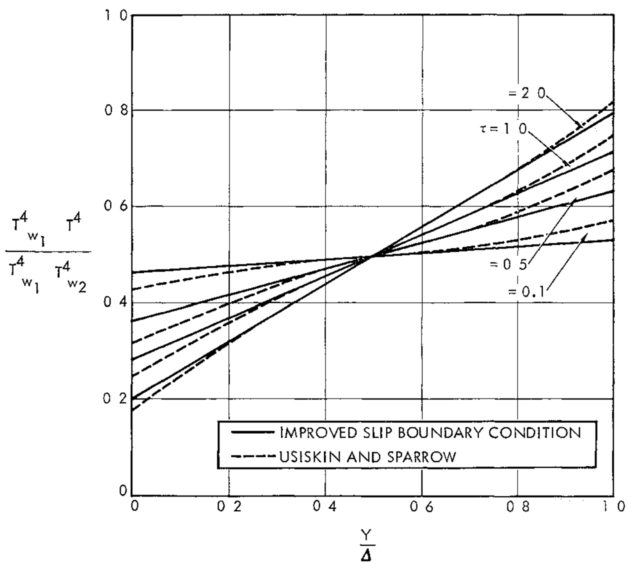

Fig 1

hold for the Rayleigh problem To demonstrate the difficulty, if the photon temperature is set equal to the gas temperature in the Rayleigh problem, and $K = \frac{2}{3}$ (as in Ref 1), the initial heat transfer is twice the blackbody value

Another limitation is imposed by the separation of the conduction and radiation heat-flow process to derive Eq (12) This separation can only be rigorously defended for the limits of a transparent or opaque gas For the transparent gas, there is no connection between the two modes of heat transfer, and the heat flux by each mode is constant across the gap For the opaque gas, when integrated with respect to temperature, the integral may be formally separated into the two terms, but the actual heat fluxes carried by radiation and conduction are not individually constant across the gap; only their sum is constant Therefore, Eq (12) must be considered a convenient expression giving the correct limits, but the justification for using it at intermediate values of τ cannot be defended by the derivation

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